

School Code: 10908



CBSE Affiliation No. : 1730578

BHAGAT PUBLIC SR. SEC. SCHOOL

ALANIYA, KOTA



MATHEMATICS

ACTIVITY

2020-21

CLASS - XII



ADDRESS
CAMPUS : N.H. 12, JHALAWAR ROAD, ALANIYA, KOTA-325003, PH :0744-2832113, 9649991123
Email : bpssschool@gmail.com

Activity \rightarrow 1

Objective: To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(L, m) | L \parallel m\}$ symmetric but neither reflexive nor transitive.

Pre-requisite knowledge: Knowledge of relations and their types, knowledge of properties of parallel and perpendicular lines.

Materials required: A thick board with board pins, some paper pins, white chart paper, some pieces of colourful threads, or yarns (blue, pen and red)

Note: Geoboard with rubber band may be used.

Procedure:

1. Place a chart paper firmly on a thick board with the help of board pins as shown in. (Fig 1.1)

2. Fix some paper pins randomly on the board as shown in. (Fig 1.2)



Fig 1.1

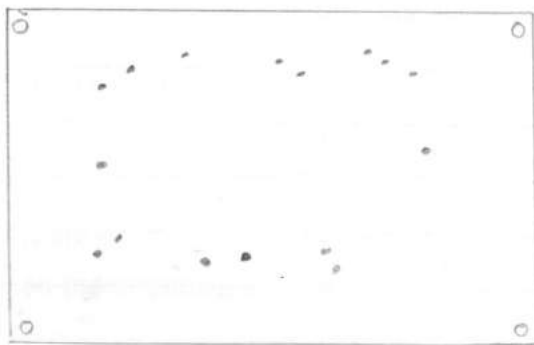


Fig 1.2



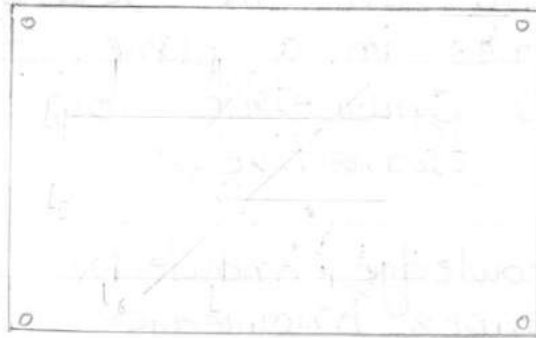
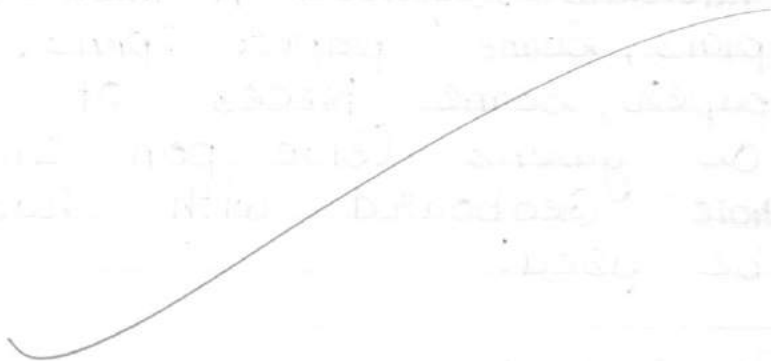


Fig. 1.3



3. Now take some pieces of thread or yarn and tie them securely around the pins making sure that there should not be any slack in the threads.

Here some perpendicular to each other and some are inclined as shown in figure 1.3.

Let the threads or yarns depict the lines L_1, L_2, \dots, L_8 . Then we have following observations.

Observations

1. L_1 is perpendicular to L_4 and L_5 .
2. L_2 is perpendicular to L_4 and L_5 .
3. L_3 is perpendicular to L_4 and L_5 .
4. L_7 is perpendicular to L_8 .
5. L_7 is perpendicular to L_6 .
6. L_4 is parallel to L_5 .
7. L_1 is parallel to L_2 , L_2 is parallel to L_3 and L_1 is parallel to L_3 .
8. L_6 is parallel to L_8 .
9. So $(L_1, L_4), (L_1, L_5), (L_2, L_4), (L_2, L_5), (L_3, L_4) \in R$
10. In fig 1.3 we see that no line is perpendicular to itself i.e. $(l, l) \notin R$ (does not belong to) R so given relation R is not reflexive.

Activity 2

Objective:- To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \parallel m\}$ an equivalence relation.

Pre-requisite knowledge: Knowledge of relations and their types namely reflexive, symmetric and transitive, equivalence relation.

Materials required: A thick board with board pins, some paper pins, white chart paper, some pieces of colourful threads or yarns (blue, green and red).
Note: Geoboard with rubber band may be used.

Procedure:

- 1 Place a chart paper firmly on a thick board with the help of board pins as shown in. (Fig 2.1)
- 2 Fix some paper pins randomly on the board as shown in fig 2.2.

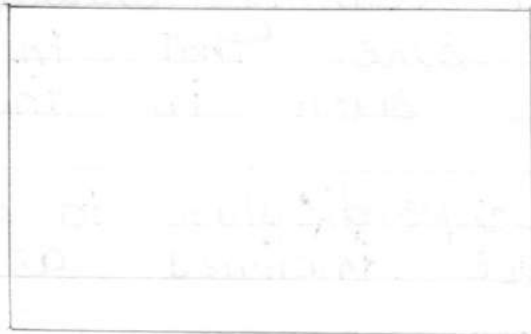


Fig 2.1

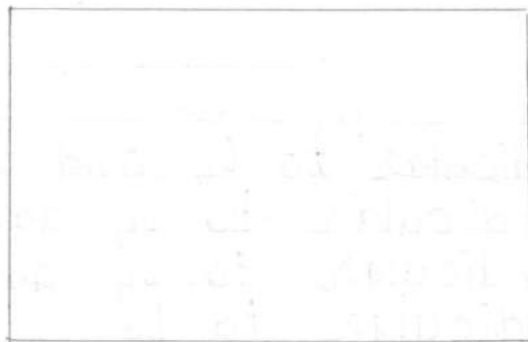


Fig 2.2



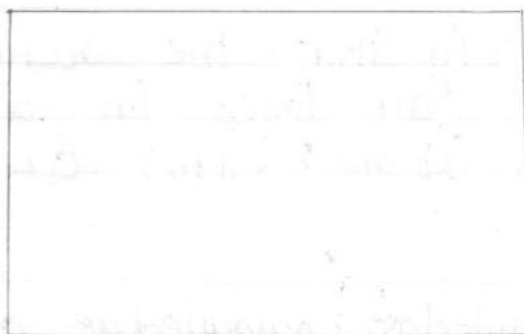
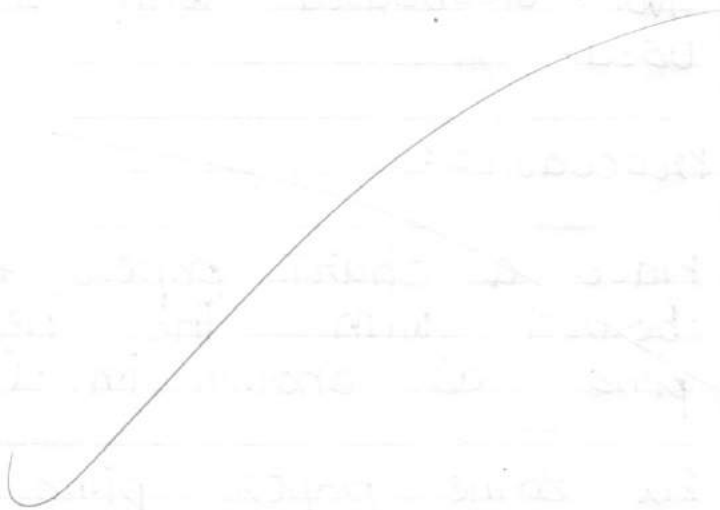


Fig 2.3



3. Now take some pieces of thread or yarn and tie them securely around the pins as shown in Fig 2.3. Note that there should not be any slack in the threads.

Here, some pieces of thread are parallel to each other, some are perpendicular and some are inclined.

Let the threads or yarns depict the lines L_1, L_2, \dots, L_8 . Then we have the following observations.

Observations

- 1 L_1 is perpendicular to L_4 and L_5 .
- 2 L_3 is perpendicular to L_4 and L_5 .
- 3 L_4 is parallel to L_5 .
- 4 L_6 is parallel to L_8 .
- 5 So $(L_4, L_5), (L_1, L_2), (L_2, L_3), (L_1, L_3), (L_6, L_8) \in R$
- 6 In fig 2.3 we see that every line is parallel to itself. Thus $L_1 \parallel L_1, L_2 \parallel L_2, \dots, L_8 \parallel L_8$.
7. In fig 2.3. we see that $L_4 \parallel L_5$ thus $L_5 \parallel L_4$.
 So $(L_4, L_5) \in R \Rightarrow (L_5, L_4) \in R$
 Similarly $(L_1, L_2) \in R \Rightarrow (L_2, L_1) \in R$
 $(L_2, L_3) \in R \Rightarrow (L_3, L_2) \in R$
 Also $(L_6, L_8) \in R \Rightarrow (L_8, L_6) \in R$.

Activity 3

Objective: To demonstrate a function which is not one-one but is onto.

Pre-requisite knowledge: Basic knowledge of relations, functions and their types such as one-one functions and onto functions.

Materials required: Coloured chart paper, a hard board, a pair of scissors, gluestick, some paper pins, some pieces of thread or yarn.

Procedure:

- 1 From a green chart paper, cut out a rectangular strip of length 15 cm and width 3 cm as shown in Fig 3.1.
- 2 From a red chart paper, cut out a rectangular strip of length 10 cm and width 3 cm as shown in Fig 3.2.

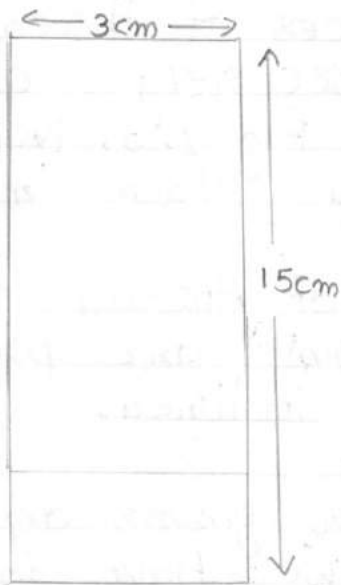


Fig 3.1



Fig 3.2

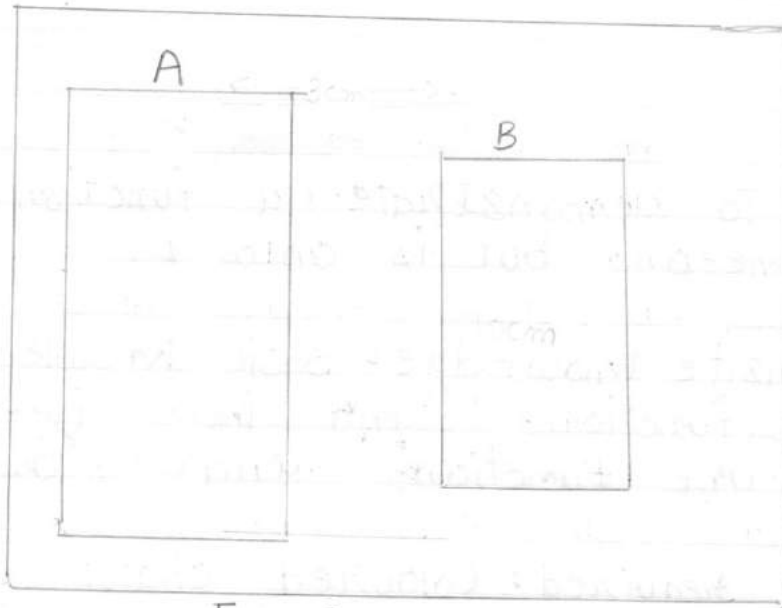


Fig 3.3

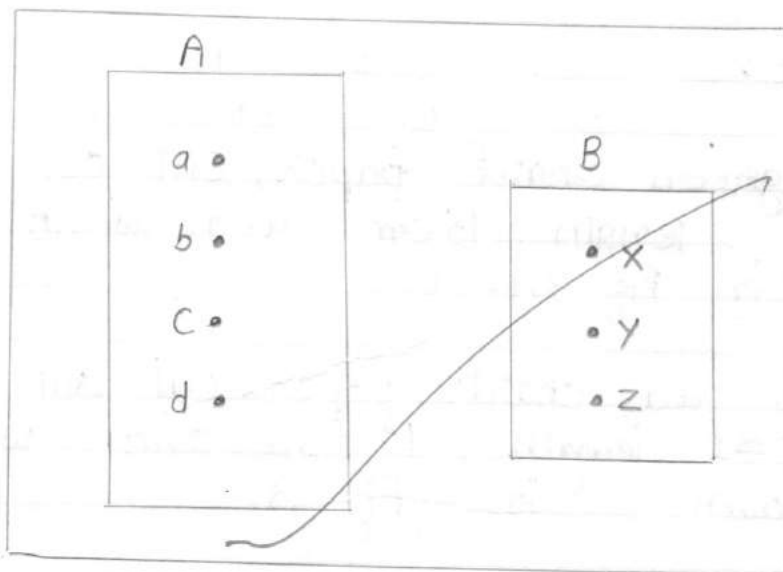


Fig 3.4

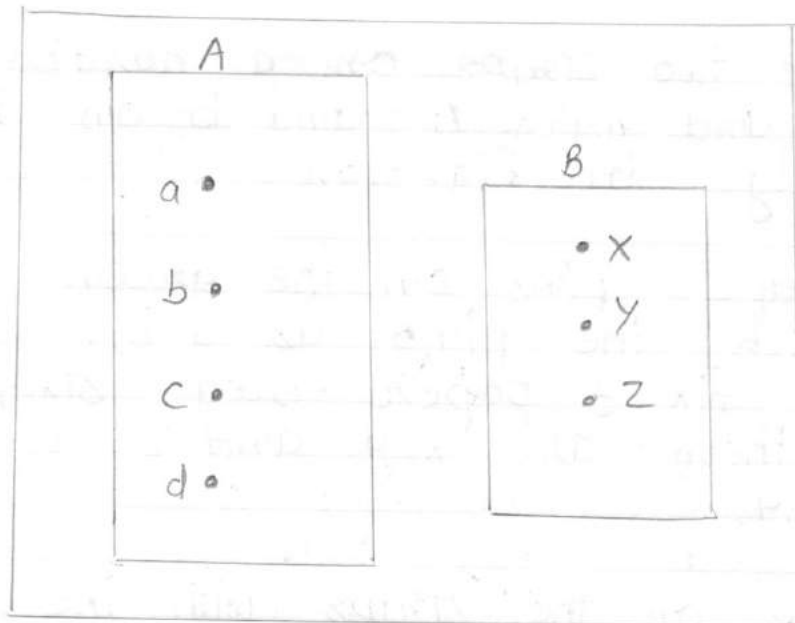


Fig 3.5

3. Paste these two strips on a hardboard side by side and mark A and B on them respectively in Fig. 3.3.

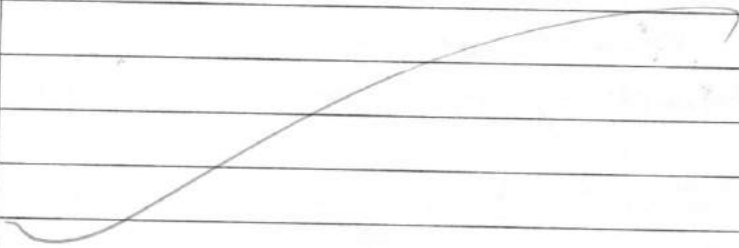
4. Fix 4 paper pins on the green strip and mark the pins as a, b, c and d. Similarly fix 3 paper red strip and mark them as x, y and z as shown in Fig 3.4.

5. Join pins on the strips with the help of threads or yarns as shown in Fig 3.5. Here keep the threads tight.

Observations

1. The image of the element a of A in B is z.
2. The image of the element b of A in B is y.
3. The image of the element c of A in B is x.
4. The image of the element d of A in B is z.
5. The pre-image of the element x of B in A is c.
6. The pre-image of the element y of B in A is b.
7. The pre-images of the element z of B in A are a and d.
8. The function shown in fig 3.5 is not one-one but onto.

9 Also, we see that every element of B is image of some element of A . So, the function is into.



Activity 4

Objective: To demonstrate a function which is one-one but not onto.

Pre-requisite knowledge: Basic knowledge of relations, functions and their types such as one-one onto-functions.

Materials required: Coloured chart paper (green, red) a hard board, a pair of (scissors), gluestick, some paper pins, some pieces of thread or yarn.

Procedure:

- 1 From a green chart paper, cut out a rectangular strip of length 15 cm and width 3 cm as shown in fig. 4.1.
- 2 From a red chart paper, cut out a rectangular strip of length 10 cm and width 3 cm as shown in fig 4.2.



Fig 4.1

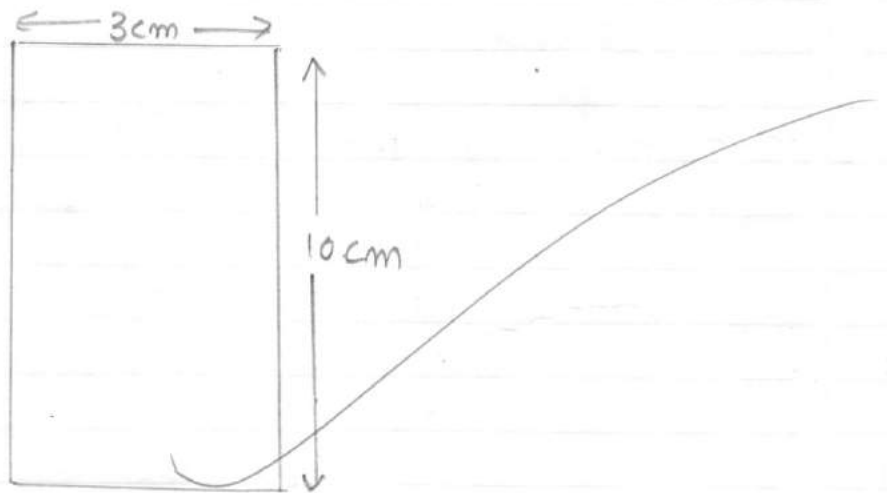


Fig 4.2

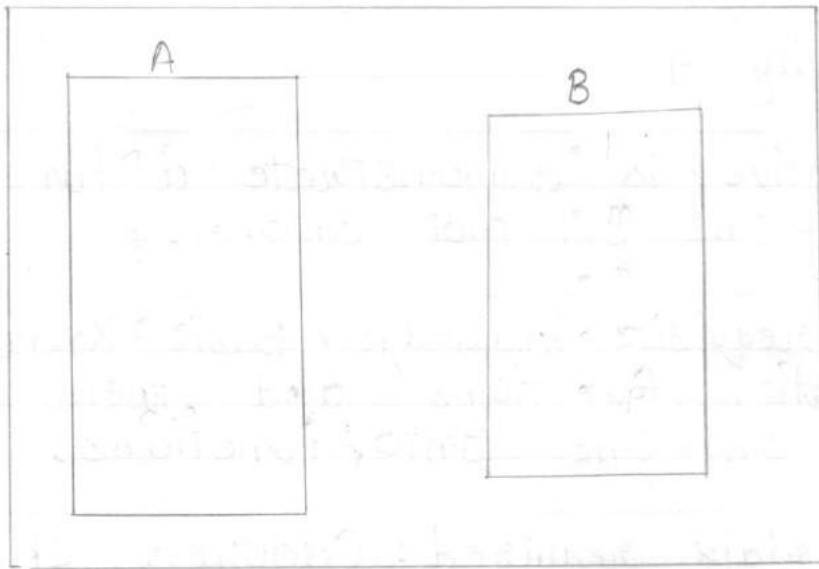


Fig 4.3

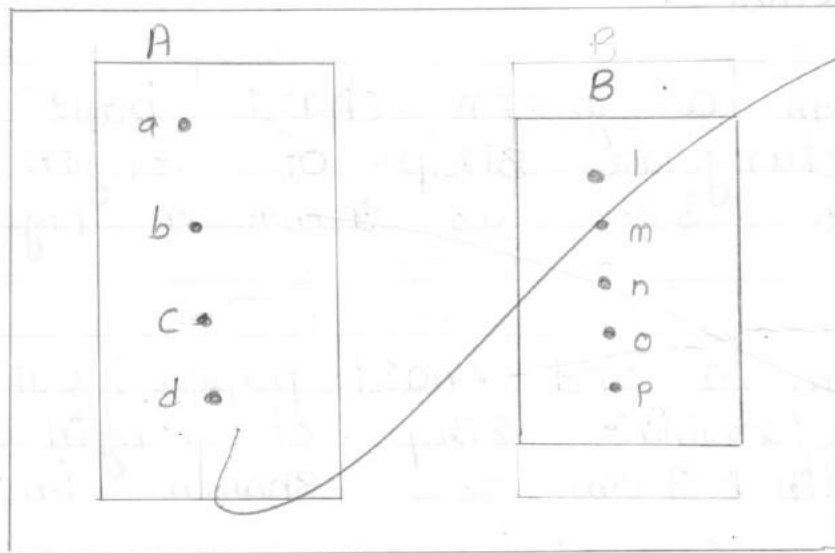


Fig 4.4

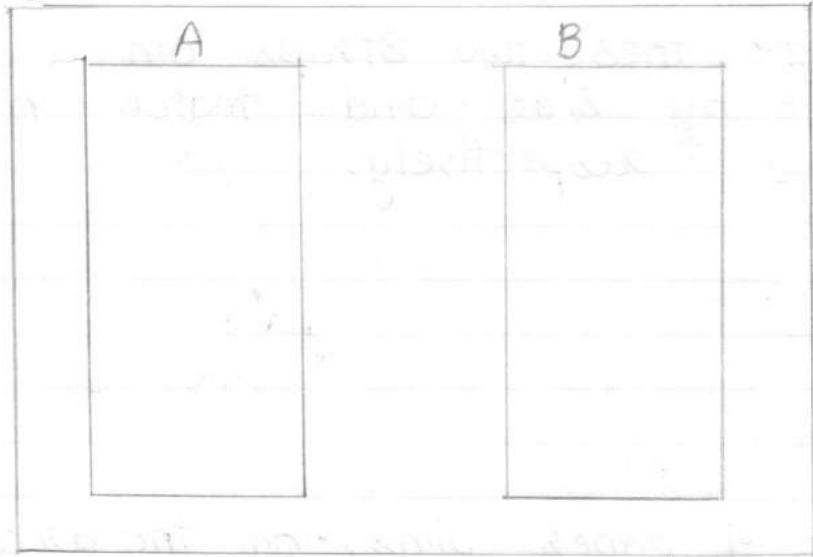
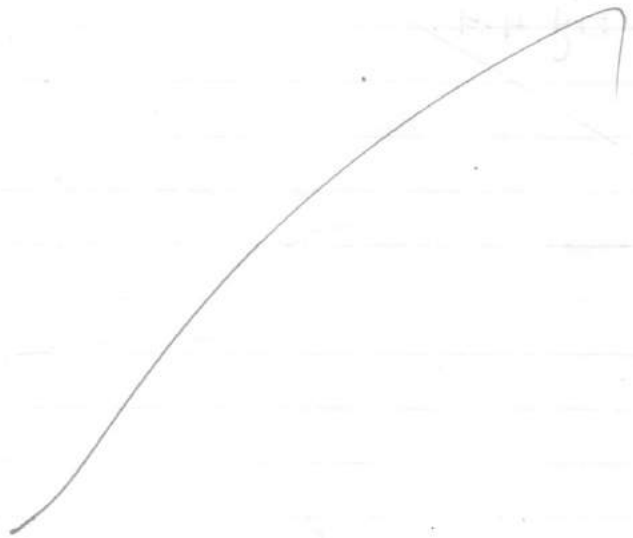


Fig 4.5



3. Paste these two strips on a hardboard side by side and mark A and B on them respectively.

4. Fix 4 paper pins on the green strip and mark the pins as a, b, c and d. Similarly fix 5 paper pins on red strip and mark them as l, m, n, o, and p, as shown in Fig 4.4.

5. Join pins on the green strip to the pins on the red strip with the help of threads or yarns as shown in Fig 4.5. Here keep the threads tight.

Observations :-

- 1 Here, Set $A = \{a, b, c, d\}$ and set $B = \{l, m, n, o, p\}$
- 2 The image of the element a of A in B is m .
- 3 The image of the element b of A in B is n .
- 4 The image of the element c of A in B is o .
- 5 The image of the element d of A in B is l .
- 6 The pre-image of $p \in B$ does not exist.
- 7 Since the images of distinct elements in A are distinct, so the function is one-one.
- 8 We also see that there is an element p in B , which has no any pre-image. So, the function onto.

Activity 5

Objective: To find analytically the limit of a function $f(x)$ at $x=c$ and also to check the continuity of the function at that point.

Pre-requisite knowledge: Knowledge of the concepts of limit and continuity of a function at a point.

Materials required: Paper, pencil, calculator.

Procedure:

1 Let the given function be $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ 4, & x = 2 \end{cases}$

2 Take some points on the left side of $c (= 2)$ which are very near to c .

3 For points on the left of $c (= 2)$, find the value of $f(x)$.

$$\Rightarrow f(x) = \frac{x^2-4}{x-2} = \frac{(x+2)(x-2)}{(x-2)} = (x+2) \quad (\because x \neq 2)$$

The values may be tabulated as below:

x	1.9	1.99	1.999	1.9999	1.99999	1.999999
f(x)	3.9	3.99	3.999	3.9999	3.99999	3.999999

4. Now, take some points on the right side of $(c=2)$, which are very near to c .

5. For points on the right of $c(=2)$, find the value of $f(x)$. The values may be tabulated as below.

x	2.1	2.01	2.001	2.0001	2.00001	2.000001
f(x)	4.1	4.01	4.001	4.0001	4.00001	4.000001

Observations

- Value of $f(x)$ is approaching to 4, as $x \rightarrow 2$ from the left.
- The value of $f(x)$ is approaching to 4, as $x \rightarrow 2$ from the right.
- So, $\lim_{x \rightarrow 2^-} f(x) = 4$ and $\lim_{x \rightarrow 2^+} f(x) = 4$
- Since, $f(2) = \lim_{x \rightarrow 2} f(x)$, So, the function is continuous at $x=2$.

Alternate Method:-

Given the function $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ 4, & x = 2 \end{cases}$ We need to check the continuity $x=2$.

Now,

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x^2-4}{x-2} = \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{(x-2)} = \lim_{x \rightarrow 2} (x+2) = 2+2=4$$

$$\text{and } f(2) = 4$$

$$\text{Thus } \lim_{x \rightarrow 2} f(x) = f(2)$$

Hence, f is continuous at $x = 2$

Activity 6

Objective: To verify Lagrange's Mean value Theorem.

Pre-requisite knowledge: knowledge of differentiability, continuity of a function and Lagrange's Mean Value Theorem.

Materials required: A piece of plywood, wires, white paper, sketch pens and wires.

Procedure:

- 1 Take a piece of Plywood of a convenient size and paste a white paper on it.
- 2 Take two wires of convenient size and paste on the white paper to represent x-axis and y-axis.
- 3 Take a piece of wire of 8 cm length and make its shape curved and paste it on the plywood shown in figure 12.1.
- 4 Take two straight wires of lengths 8 cm and 11 cm and fix them at two different points M and the curve parallel to y-axis and their feet touching the x-axis at A and B respectively. Join point and N using a wire.

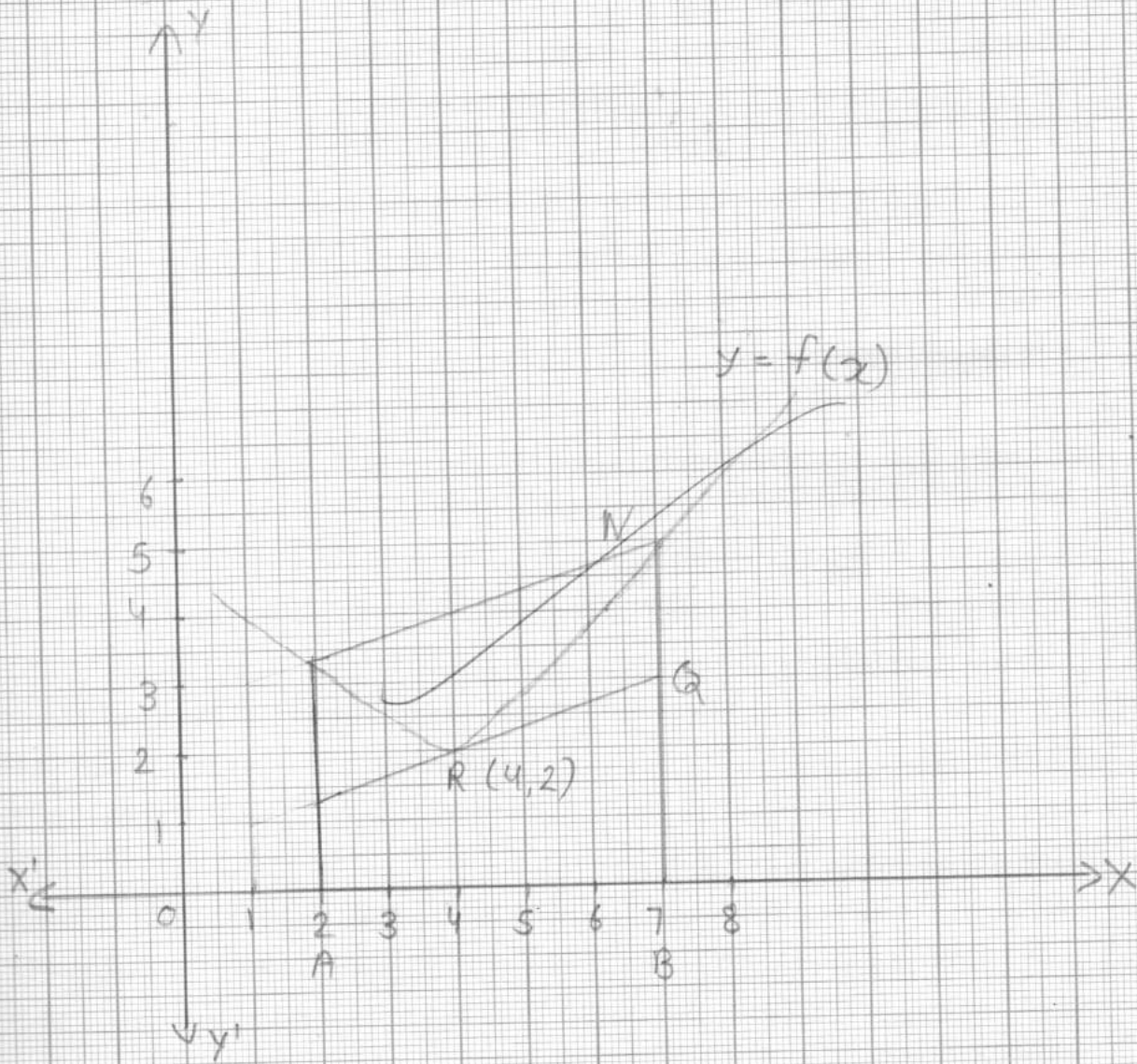


Fig 12.1

Observations

- 1 In the figure, the curve represents the function $y = f(x)$.
Here $OA = 2$ units and $OB = 7$ units
- 2 The co-ordinates of the points A and B are $(2, 0)$ and $(7, 0)$ respectively.
- 3 $f'(2)$ is the slope of the tangent PQ at $x = 2$.
- 4 The slope of the chord MN is $\frac{5-3}{7-2} = 0.4$
- 5 $MN \parallel PQ$ Therefore $f'(2) = 0.4$
Hence the Lagrange's Mean Value Theorem is verified.

Activity 7

Objective: To understand the concepts of decreasing and increasing function.

Pre-requisite knowledge: Knowledge of increasing and decreasing functions and their basic property.

Materials required: Pieces of wire of different lengths, piece of plywood of suitable size, white paper, glue stick, trigonometric tables or scientific calculator.

Procedure:

1. Take a piece of plywood of a convenient size and paste a white paper on it.
2. Take two pieces of wire, each of suitable length and fix them on the white paper to represent x and y -axis.
3. Take two more pieces of wire, each of suitable length and bend them in the shape of curves representing two functions and fix them on the paper to represent two functions as shown in figure 13.1.

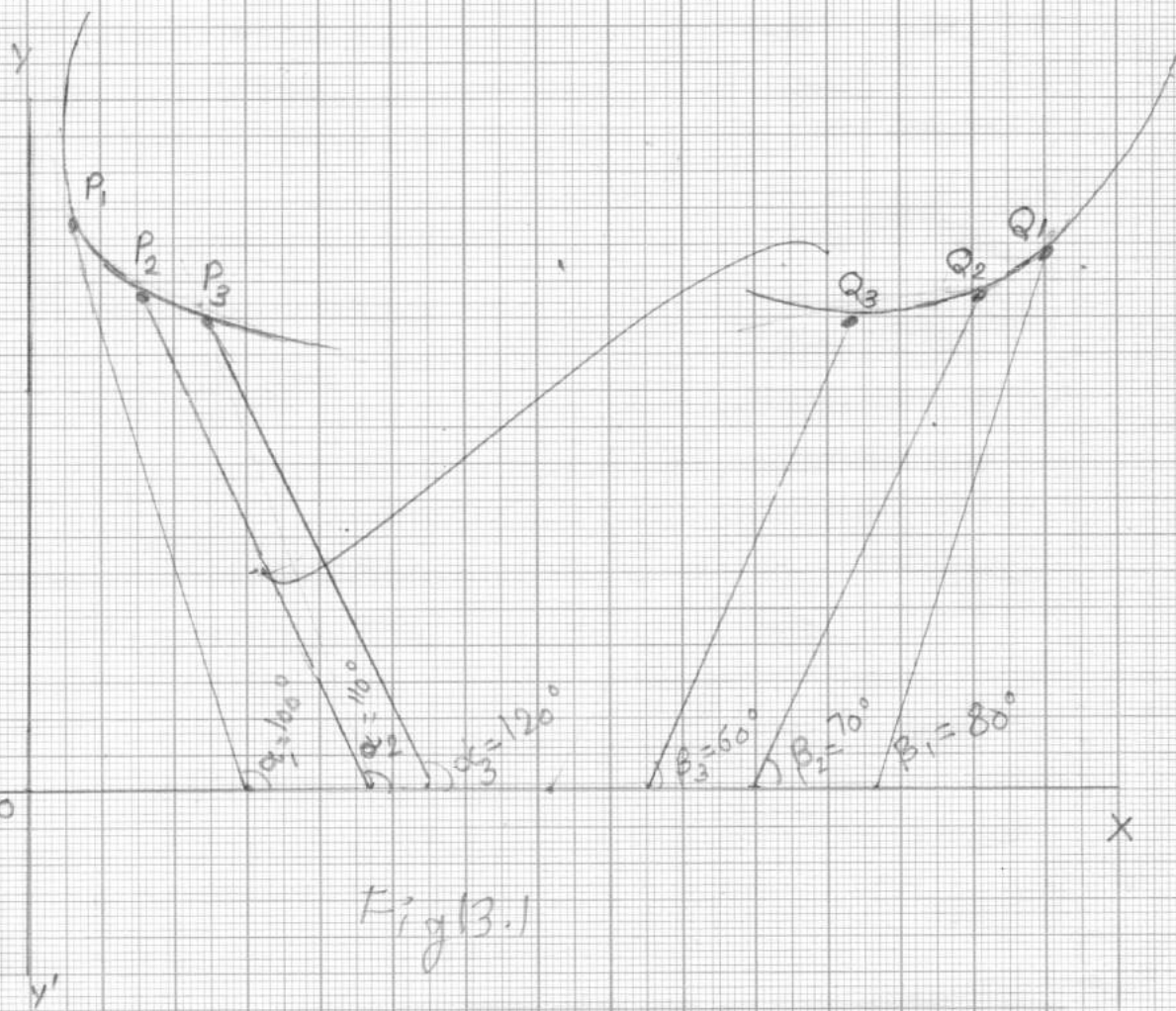


Fig 13.1

4. Take one straight wire and place it on the curve such that it is tangent to the curve point say P_1 and making an angle α_1 with the positive direction of x-axis.
5. Now take three points Q_1, Q_2, Q_3 on the curve and using the other straight wires, form tangents at each of these points making angles $\beta_1, \beta_2, \beta_3$ respectively with the positive direction of x-axis as shown in figure 13.1.

Observations

- $\alpha_1 = 100^\circ$ is an obtuse angle, so $\tan \alpha_1$ is negative, i.e., the slope of the tangent at P_1 is negative.
- Again $\alpha_2 = 110^\circ$ and $\alpha_3 = 120^\circ$ are obtuse angles and therefore, slopes of the tangents $\tan \alpha_2$ and $\tan \alpha_3$ are both negative.
- $\Rightarrow \tan \alpha_1 = \tan 100^\circ = -5.6712$
 $\Rightarrow \tan \alpha_2 = \tan 110^\circ = -2.7474$
 $\Rightarrow \tan \alpha_3 = \tan 120^\circ = -1.7320$
- The function given by the curve is a decreasing function.

Activity 8

Objective: To understand the concepts of absolute maximum and minimum values of a function in a given closed interval through its graph.

Pre-requisite knowledge: Knowledge of maxima, minima, absolute maximum, absolute minimum etc.

Materials required: Drawing board, white chart paper, adhesive, sketch pens, calculator.

Procedure:

1. Paste a white chart paper of suitable size on the drawing board.
2. Draw two lines on the graph paper representing the two rectangular axes, as shown in the figure 15.1.
3. Graduate the two axes as shown in the figure 15.1.
4. Let us consider $f(x) = 2x^3 + 3x^2 - 12x$ in the interval $[-3, 3]$
5. We take different values of x in $[-3, 3]$, find the values of $f(x)$

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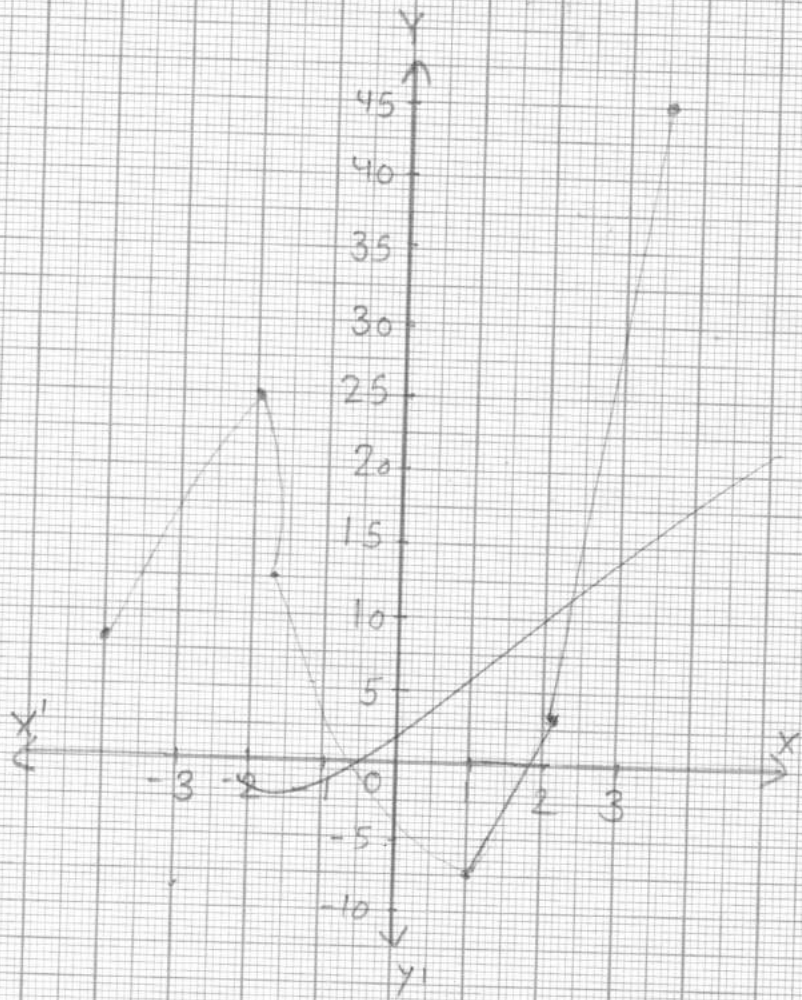


Fig 15.1

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Some ordered pairs representing points on the graph of $f(x)$ are as follows:

x	-3	-2	-1	0	1	2	3
$f(x)$	9	20	13	0	-7	4	45

We join the plotted points by a free hand curve and obtain the graph of the function as shown in figure 15.1

Observations

From the graph, we see that the value of $f(x)$ at $x=3$ is 45, which is maximum in $[-3, 3]$. So, absolute maximum value of $f(x)$ is 45.

Also, from the graph, we see that the value of $f(x)$ at $x=1$ is -7, which is minimum in $[-3, 3]$. So, absolute minimum value of $f(x)$ is -7.

Activity 9

Objective: To construct an open box of maximum volume from a given rectangular sheet by squares from each corner.

Pre-requisite knowledge: Knowledge of properties of cuboid and formula of its volume.

Materials required: Chart papers, scissors, cellotape, gluestick and calculator.

Procedure:

1. Take a rectangular chart paper $EFGH$ of size $45\text{cm} \times 24\text{cm}$.
2. Cut four equal squares each of side $x\text{cm}$ from each corner E, F, G and H .
3. Repeat the activity by taking the same size of chart papers and different values of x .
4. Construct an open box by folding the flaps of the sheets cut using cellotape/glue stick.

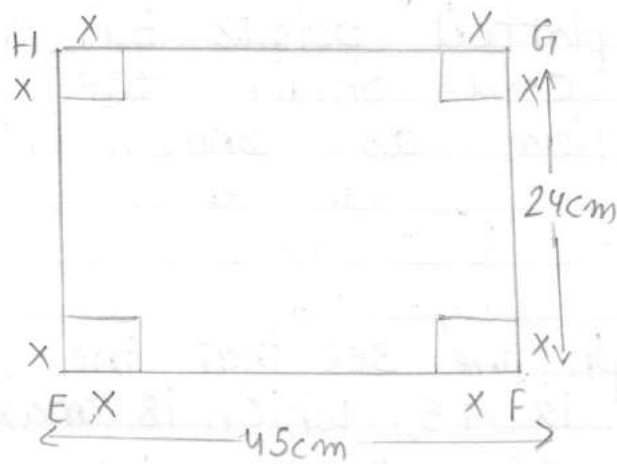


Figure 16.1

Observations

- 1 When $x = 1$ cm, Volume of the open box $= (45-2) \times (24-2) \times 1 \text{ cm}^3 = 946 \text{ cm}^3$
- 2 When $x = 2$ cm, Volume of the open box $= (45-4) \times (24-2) \times 2 \text{ cm}^3 = 1640 \text{ cm}^3$
- 3 When $x = 3$ cm, Volume of the open box $= (45-6) \times (24-6) \times 3 \text{ cm}^3 = 2106 \text{ cm}^3$
- 4 When $x = 4$ cm, Volume of the open box $= (45-8) \times (24-8) \times 4 \text{ cm}^3 = 2368 \text{ cm}^3$
- 5 When $x = 5$ cm, Volume of the open box $= (45-10) \times (24-10) \times 5 \text{ cm}^3 = 2450 \text{ cm}^3$

Here we observe that when $x = 5$ cm, Volume of the open box is maximum.

Activity 10

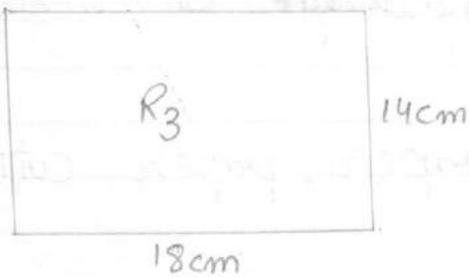
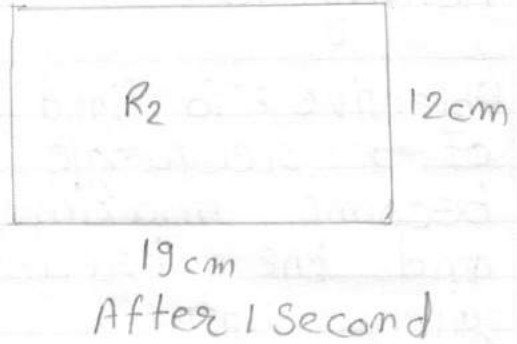
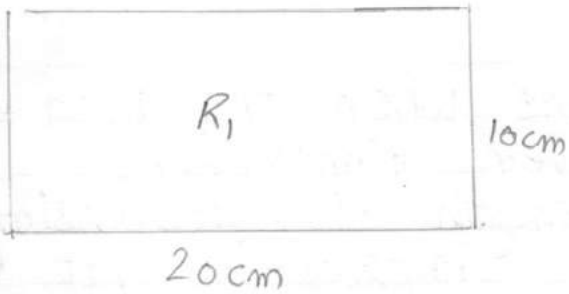
Objective: To find the time when the area of a rectangle of given dimensions become maximum length is decreasing and the breadth is increasing at given rates.

Pre-requisite knowledge: knowledge of rectangles and their areas.

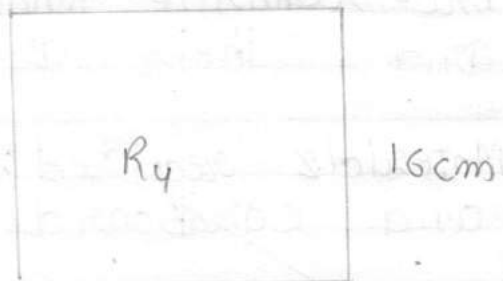
Materials required: Chart paper, paper cutter and cardboard.

Procedure:

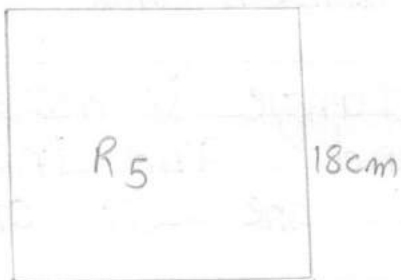
- 1 We take a rectangle R_1 of dimensions $20\text{cm} \times 10\text{cm}$.
- 2 Let the length of the rectangle is decreasing at the rate of $1\text{cm}/\text{second}$ and the breadth is increasing at the rate of $2\text{cm}/\text{second}$.
- 3 Cut other rectangles $R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9$ and R_{10} of dimensions $19\text{cm} \times 12\text{cm}, 14\text{cm}, 17\text{cm} \times 16, 16\text{cm} \times 18\text{cm}, 15\text{cm} \times 20\text{cm}, 14\text{cm} \times 22\text{cm}, 13\text{cm} \times 24\text{cm} \times 26\text{cm}$ and $11\text{cm} \times 28\text{cm}$ respectively as shown in figure 17.1.



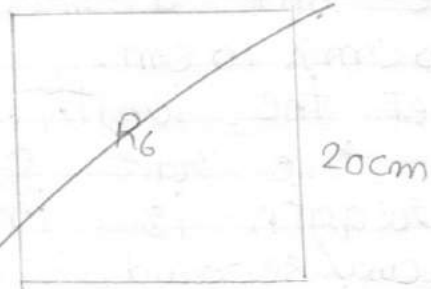
After 2 seconds



After 3 seconds



After 4 seconds



After 5 seconds

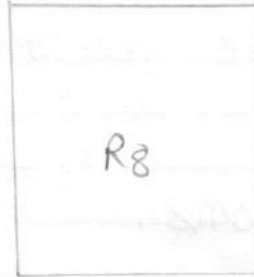


R_7

22cm

14cm

After 6 seconds



R_8

24cm

13cm

After 7 seconds

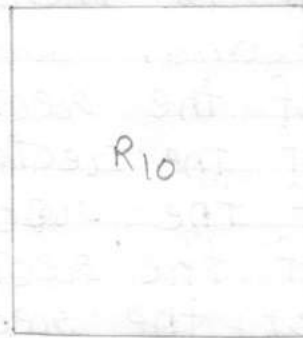


R_9

26cm

12cm

After 8 seconds



R_{10}

28cm

11cm

After 9 seconds



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Principal
Bhagat Public Sr. Sec. School
Alaniya, Alaniya, Haryana

4. Fix^o these rectangles on a cardboard.

Observations

- 1 Length of the rectangle is decreasing at the rate of 1 cm/second and the breadth is increasing at the rate of 2 cm/second .
- 2 Area of the rectangle, $R_1 = 20 \times 10\text{ cm}^2 = 200\text{ cm}^2$
- 3 Area of the rectangle, $R_2 = 19 \times 12\text{ cm}^2 = 228\text{ cm}^2$
- 4 Area of the rectangle, $R_3 = 18 \times 14\text{ cm}^2 = 252\text{ cm}^2$
- 5 Area of the rectangle, $R_4 = 17 \times 16\text{ cm}^2 = 272\text{ cm}^2$
- 6 Area of the rectangle $R_5 = 16 \times 18\text{ cm}^2 = 288\text{ cm}^2$
- 7 Area of the rectangle $R_6 = 15 \times 20\text{ cm}^2 = 300\text{ cm}^2$
- 8 Area of the rectangle $R_7 = 14 \times 22\text{ cm}^2 = 308\text{ cm}^2$
- 9 Area of the rectangle $R_8 = 13 \times 24\text{ cm}^2 = 312\text{ cm}^2$
- 10 Area of the rectangle $R_{10} = 11 \times 28\text{ cm}^2 = 308\text{ cm}^2$
- 11 Hence, maximum area of the rectangle is 312 cm^2 .